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## INDIAN SCHOOL SALALAH

FIRST TERM EXAMINATION – SEPTEMBER (AY- 2024-25)



Class: XII

MATHEMATICS (041)

Date: 30/09/2024

Time: 3 hours

Maximum Marks: 80

## General Instructions:

- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

NO	SECTION A	MARKS
1	A function $f: Z \rightarrow Z$ given by $f(x) = 5x + 3$ is (a) one-one but not onto. (b) bijective (c) onto but not one-one (d) None of these	1
2	Principal value of $\cot\left(\frac{\pi}{2} - \cot^{-1}\sqrt{3}\right)$ (a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 2 (d) 1	1
3	If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ , then write the value of $x$ and $y$ . (a) $x = 3, y = 3$ (b) $x = 3, y = 2$ (c) $x = 2, y = 2$ (d) $x = 2, y = 3$	1
4	If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$ , then the value of $x$ is: (a) 6 (b) 3 (c) 7 (d) 1	1

5	If $y = Ae^{5x} + Be^{-5x}$ then $\frac{d^2y}{dx^2}$ is equal to (a) $25y$ (b) $5y$ (c) $-25y$ (d) $10y$	1
6	The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval (a) $(2, \infty)$ (b) $(-\infty, 2)$ (c) on $\mathbb{R}$ (d) None of the above	1
7	Evaluate: $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ (a) $\tan x - \cot x + C$ (b) $-\tan x + \cot x + C$ (c) $\tan x + \cot x + C$ (d) $-\tan x - \cot x + C$	1
8	Let set $X = \{1, 2, 3\}$ and a relation $R$ is defined in $X$ as : $R = \{(1, 3), (2, 2), (3, 2)\}$ , then minimum ordered pairs which should be added in relation $R$ to make it reflexive and symmetric are (a) $\{(1, 1), (2, 3), (1, 2)\}$ (b) $\{(3, 3), (3, 1), (1, 2)\}$ (c) $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$ (d) $\{(1, 1), (3, 3), (3, 1), (1, 2)\}$	1
9	Which of the following relations is symmetric and transitive but not reflexive for the set $I = \{4, 5\}$ ? (a) $R = \{(4, 4), (5, 4), (5, 5)\}$ (b) $R = \{(4, 4), (5, 5)\}$ (c) $R = \{(4, 5), (5, 4)\}$ (d) $R = \{(4, 5), (5, 4), (4, 4)\}$	1
10	The matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is a symmetric matrix. Then the value of $a$ and $b$ respectively are: (a) $\frac{-2}{3}, \frac{3}{2}$ (b) $\frac{-1}{2}, \frac{1}{2}$ (c) $-2, 2$ (d) $\frac{3}{2}, \frac{1}{2}$	1
11	If $A$ is a non-singular matrix of order 3 and $ A  = -4$ , find $ \text{adj } A $ . (a) 4 (b) 16 (c) 64 (d) $\frac{1}{4}$	1
12	If $y = \sqrt{a^2 - x^2}$ , then $y \frac{dy}{dx}$ is: (a) 0 (b) $x$ (c) $-x$ (d) 1	1
13	The rate of change of volume of a cylinder w.r.t. radius whose radius is equal to its height is (a) 4 (area of base) (b) 3 (area of base) (c) 2 (area of base) (d) (area of base)	1

14	$\int x \log x dx =$ (a) $\frac{x^2}{2} \log x - x \log x + 2x - C$ (b) $\frac{x^2}{2} \log x + x \log x + x + C$ (c) $\frac{x^2}{2} \log x - x \log x - x + C$ (d) $\frac{x^2}{2} \log x - \frac{x^2}{4} + C$	1
15	If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$ , find $x$ . (a) 13      (b) 3      (c) -13      (d) $\sqrt{3}$	1
16	If $f(x) = x \tan^{-1} x$ , then $f'(1) =$ (a) $1 + \frac{\pi}{4}$ (b) $\frac{1}{2} + \frac{\pi}{4}$ (c) $\frac{1}{2} - \frac{\pi}{4}$ (d) 2	1
17	The rate of change of the radius of a sphere is $1/2\pi$ . When it's radius is 5cm, what will be the rate of change of the surface area of the sphere? (a) 10 sq cm      (b) 20 sq cm      (c) 30 sq cm      (d) 40 sq cm	1
18	$\int \cos^3 x \cdot e^{\log(\sin x)} dx$ is equal to (a) $-\frac{\cos^4 x}{4} + C$ (b) $-\frac{\sin^4 x}{4} + C$ (c) $\frac{e^{\sin x}}{4} + C$ (d) none of these	1
19	<b>Assertion (A):</b> We can write $\sin^{-1} x = (\sin x)^{-1}$ . <b>Reason (R):</b> Any value in the range of principal value branch is called principal value of that inverse trigonometric function. (a) Both A and R are true and R is the correct explanation of A . (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true	1
20	<b>Assertion (A):</b> The matrix $A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$ is a skew symmetric matrix. <b>Reason (R):</b> For the given matrix A we have $A' = A$ . (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true	1

	<b>SECTION B</b>	
21	Express $\tan^{-1}\left(\frac{\sin x}{1-\cos x}\right)$ in simplest form.	2
22	Find the values of k, if area of triangle is 4 sq. units whose vertices are (k, 0), (4,0) and (0,2) respectively.	2
23	<p>For what value of 'k' is the function <math>f(x) = \begin{cases} \frac{\sin 5x}{3x} + \cos x, &amp; \text{if } x \neq 0 \\ k, &amp; \text{if } x = 0 \end{cases}</math> continuous at <math>x = 0</math> ?</p> <p style="text-align: center;"><b>OR</b></p> <p>Discuss the continuity of the following function at <math>x = 0</math> :</p> $f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$	2
24	If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ , then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$	2
25	<p>Evaluate <math>\int \frac{dx}{x(1+\log x)^2}</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>Find <math>\int \frac{\sin^6 x}{\cos^8 x} dx</math></p>	2
	<b>SECTION C</b>	
26	<p>Find the value of k so that the function <math>f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, &amp; \text{if } x \neq \frac{\pi}{2} \\ 3, &amp; \text{if } x = \frac{\pi}{2} \end{cases}</math> is continuous at point <math>x = \pi/2</math></p> <p style="text-align: center;"><b>OR</b></p> <p>If <math>x \sin(a+y) + \sin a \cos(a+y) = 0</math>, then prove that <math>\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}</math></p>	3
27	Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.	3
28	If $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix} = 8$ , then find the x value.	3
29	<p>Find the intervals in which <math>f(x) = -2x^3 - 9x^2 - 12x + 1</math> is strictly increasing or decreasing function.</p> <p style="text-align: center;"><b>OR</b></p>	3

	Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$ , where $x$ is the number of units and $P$ is the profit in rupees.	
30	<p>Evaluate <math>\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Evaluate: <math>\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx</math></p>	3
31	Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $A^2 - 6A + 17I = O$ . Hence find $A^{-1}$ .	3
	<b>SECTION D</b>	
32	<p>If <math>A = \begin{bmatrix} 2 &amp; -3 &amp; 5 \\ 3 &amp; 2 &amp; -4 \\ 1 &amp; 1 &amp; -2 \end{bmatrix}</math>, find <math>A^{-1}</math> and hence solve the system of linear equations:</p> <p><math>2x - 3y + 5z = 11</math>; <math>3x + 2y - 4z = -5</math>; <math>x + y - 2z = -3</math></p>	5
33	<p>Let <math>N</math> denote the set of all natural numbers and <math>R</math> be the relation on <math>N \times N</math> defined by <math>(a, b)R(c, d)</math> if <math>ad(b + c) = bc(a + d)</math>. Show that <math>R</math> is an equivalence relation.</p> <p style="text-align: center;"><b>OR</b></p> <p>Let <math>A = \mathbb{R} - \{3\}</math> and <math>B = \mathbb{R} - \{1\}</math>. Prove that the function <math>f: A \rightarrow B</math> defined by <math>f(x) = \left(\frac{x-2}{x-3}\right)</math> is one-one and onto? Justify your answer.</p>	5
34	Show that the semi-vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1} \sqrt{2}$ .	5
35	<p>Evaluate: <math>\int \frac{(x^2-3x)}{(x-1)(x-2)} dx</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Evaluate: <math>\int \frac{(x^2+1)(x^2+2)}{(x^2+4)(x^2+3)} dx</math></p>	5
	<b>SECTION E</b>	
36	<p><b>Case Study .1:</b></p> <p>A craftsman made a Doll toy, where the height of the doll is based on the function <math>f(x) =  x - 3  +  x + 2 </math>. Based on this information answer the followings:</p>	



- a) When  $x < -2$ , what will be the height in terms of  $x$ ? 1
- b) When the  $x$  value lies between  $(-2,3)$  then find the function? 1
- c) Will the slope vary with  $x$  value if  $x$  value lies between  $(-2,3)$ ? Why? 2

**OR**

- c) If the craftsman is trying to make a doll using the function  $f(x) = [x]$ , is he able to make? Why?

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**Case Study .2:**

Ram purchases 3 pens, 2 bags, and 1 instrument box and pays ₹ 41. From the same shop, Dheeraj purchases 2 pens, 1 bag, and 2 instrument boxes and pays ₹29, while Ankur purchases 2 pens, 2 bags, and 2 instrument boxes and pays ₹44.



- a) What is the total cost of one pen and one bag? 1
- b) What is the total cost of one pen & one instrument box? 1
- c) What is the total cost of one bag & one instrument box? 2

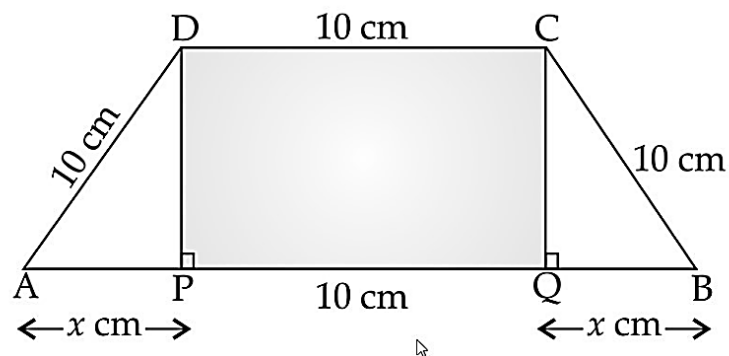
**OR**

- c) Find the total cost of one pen, one bag, and one instrument box.

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**Case Study .3:**

There is a bridge whose length of three sides of trapezium other than base are equal to 10 cm



- Find area  $A(x)$  of trapezium in terms of  $x$ .
- Find the value of  $x$ , if  $A'(x) = 0$ .

2

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