MATHEMATICS   MARKS: 100

CLASS: XI   TIME: 3HRS

General Instructions:

i. All questions are compulsory.

ii. This question paper contains 29 questions.

iii. Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.

iv. Question 5-12 in Section B are short-answer type questions carrying 2 marks each.

v. Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.

vi. Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section. A

Questions 1 to 4 carry 1 mark each.

1. Find the value of $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$

2. Find the derivative of $\dfrac{\sin x}{1 - x^2}$

3. Find the multiplicative inverse of $\sqrt{5} + 3i$.

4. Write the negation of the compound statement 'Square of an integer is positive or negative'.

Section.B

Questions 5 to 12 carry 2 marks each.

5. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

6. Evaluate : $\lim\limits_{x \to 2} \dfrac{3x^2 - x + 10}{x^2 - 4}$

7. Find the modulus of $\dfrac{2 + i}{4i + (1 + i)^2}$

8. Find equation of the line perpendicular to the line $3x - 4y + 2 = 0$ and passing through the point (–2, 3).

9. Verify that (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.

10. If $^{20}P_r = 13 \times {}^{20}P_{r-1}$, find r.

11. Find all pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

12. The first term of a G.P is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.

Section.C

Questions 13 to 23 carry 4 marks each.

13. In a survey of 70 people, it was found that 29 people read newspaper H, 31 read newspaper T, 28 read newspaper M, 9 read both H and M, 11 read both H and T, 8 read both T and M, 3 read all three newspapers.

    a) Find the number of people who read at least one of the newspapers.

    b) Find the number of people who read exactly one newspaper.

    c) What values are depicted in these people's activity?

14. Find the derivative of $\tan x$ from first principle.

    OR

    Find the derivative of $\dfrac{2}{x+1}$ from first principle.

15. Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

16. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

17. Prove that $(b+c)\cos\dfrac{B+C}{2} = a\cos\dfrac{B-C}{2}$

    OR

    Prove that $a(b\cos C - c\cos B) = b^2 - c^2$

18. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

19. Find the square root of $-15 - 8i$

20. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?

21. The sum of two numbers is 6 times their geometric means, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

    OR

    Find the sum to *n* terms of the series 1 x 2 x 3 + 2 x 3 x 4 + 3 x 4 x 5 + …….

22. Find the coordinates of the foci, the vertices, the length of major axis and the eccentricity of the ellipse $9x^2 + 4y^2 = 36$

23. Find the mean deviation about the median for the following data:

x_i	3	6	9	12	15	17
f_i	2	4	5	2	3	2

Section.D

Questions 24 to 29 carry 6 marks each.

24. a) Solve: $2\cos^2\theta + 3\sin\theta = 0$

    b) Prove that $\sin\theta + \sin(\theta + \frac{2\pi}{3}) + \sin(\theta + \frac{4\pi}{3}) = 0$

    OR

    a) Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

    b) Prove that $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

25. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. But later found that two observations were taken wrongly as 8 and 10 instead of 12 and 16. Find the correct mean and standard deviation.

26. By using principle of mathematical induction prove that

    $$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}, \forall n \in N$$

27. Expand by using Binomial Theorem $\left(1 + \dfrac{x}{2} - \dfrac{2}{x}\right)^4$

    OR

    The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^n$ are in the ratio 1:3:5. Find n and r.

28. Prove the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ is b^2.

    OR

    Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

29. Solve the system of inequalities graphically:

    $3x + 2y \le 90$, $x + 4y \le 100$, $x \ge 20$, $y \ge 0$

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